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Perturbing around a warped product of AdS_4 and seven-ellipsoid

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Abstract: We compute the spin-2 Kaluza-Klein modes around a warped product of AdS_4 and a seven-ellipsoid. This background with global G_2 symmetry is related to a U(N) × U(N) $\mathcal{N} = 1$ superconformal Chern-Simons matter theory with sixth order superpotential. The mass-squared in AdS_4 is quadratic in G_2 quantum number and KK excitation number. We determine the dimensions of spin-2 operators using the AdS/CFT correspondence. The connection to $\mathcal{N} = 2$ theory preserving SU(3) × U(1)$_R$ is also discussed.

Keywords: AdS-CFT Correspondence, M-Theory

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Contents

1 Introduction

The three-dimensional $\mathcal{N} = 6$ $\mathrm{U}(N) \times \mathrm{U}(N)$ Chern-Simons matter theories with level k can be described as the low energy limit of N M2-branes at $\mathbf{C}^4/\mathbf{Z}_k$ singularity [1]. For $k = 1, 2$, the full $\mathcal{N} = 8$ supersymmetry is preserved while for $k > 2$, the supersymmetry is broken to $\mathcal{N} = 6$. The RG flow between the UV fixed point and the IR fixed point of the three-dimensional field theory can be found from gauged $\mathcal{N} = 8$ supergravity in four-dimensions via AdS/CFT correspondence [2]. The holographic RG flow equations connecting $\mathcal{N} = 8$ SO(8) fixed point to $\mathcal{N} = 2$ $\mathrm{SU}(3) \times \mathrm{U}(1)$ fixed point have been found in [3, 4] while those from $\mathcal{N} = 8$ SO(8) fixed point to $\mathcal{N} = 1$ G_2 fixed point also have been studied in [4–6]. The M-theory lifts of these RG flows have been constructed in [5, 7].

The mass deformed $\mathrm{U}(2) \times \mathrm{U}(2)$ Chern-Simons matter theory with level $k = 1$ or $k = 2$ preserving global $\mathrm{SU}(3) \times \mathrm{U}(1)_R$ symmetry has been studied in [8–11] while the mass deformation for this theory preserving G_2 symmetry has been described in [12]. The nonsupersymmetric RG flow equations preserving $\mathrm{SO}(7)^{\pm}$ symmetry have been discussed in [13]. The holographic RG flow equations connecting $\mathcal{N} = 1$ G_2 fixed point to $\mathcal{N} = 2$ $\mathrm{SU}(3) \times \mathrm{U}(1)_R$ fixed point have been found in [14]. Moreover, the $\mathcal{N} = 4$ and $\mathcal{N} = 8$ RG flows have been studied in [15]. Recently, further developments on the gauged $\mathcal{N} = 8$ supergravity in four-dimensions have been done in [16, 17].

In order to understand the above $\mathcal{N} = 1$ mass-deformed Chern-Simons matter theory preserving G_2 symmetry(no R-symmetry and no chiral ring) in three-dimensions fully, the gravity dual should be used to study this strongly coupled field theory and this is the main feature of AdS/CFT correspondence [2]. Heidenreich [18] found the complete list of $OSp(1|4)$ unitary irreducible representations: the structure of $\mathcal{N} = 1$ supermultiplets. The even subalgebra of $OSp(1|4)$ is the isometry algebra of AdS_4. In his classification, there exist massive supermultiplets. Then the $\mathcal{N} = 1$ supermultiplets of gravity states of IR theory at the first few KK levels can be classified according to the mass spectrum.

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The 4-dimensional KK modes are massive AdS_4 scalars and the masses are determined by the eigenvalues of the differential operator acting on 7-dimensional ellipsoid. Then it is necessary to compute this eigenvalue equation, i.e., both the eigenfunctions and the eigenvalues for given above 7-dimensional Laplacian.

In this paper, we compute the explicit KK spectrum of the spin-2 fields in AdS_4 by following the recent work of [11]. In an 11-dimensional theory, the equations for the metric perturbations leads to a minimally coupled scalar equation. We obtain all the KK modes that are polynomials in the eight variables parametrizing the deformed $\mathbf{R}^8$. The squared-mass terms in AdS_4 for all the modes are quadratic of the G_2 quantum number and the KK excitation number. We describe the corresponding $\mathcal{N}=1$ dual SCFT operator depending on the two quantum numbers.

In section 2, we review the 11-dimensional background discovered in [5]. In section 3, we solve the minimally coupled scalar equation in this background and its spectrum is determined. We match the quantum numbers of the operators with those of the operators in Chern-Simons matter theory with sixth order superpotential. In section 5, we summarize the main results of this paper and make some comments on the future directions.

2 An $\mathcal{N}=1$ supersymmetric G_2-invariant flow in M-theory: review

Let us review the 11-dimensional uplift of the supergravity background with global G_2 symmetry found in [19] as a nontrivial extremum of the gauged $\mathcal{N}=8$ supergravity in 4-dimensions. Our notation is as follows: the 11-dimensional coordinates with indices $M, N, \cdots$ are decomposed into 4-dimensional spacetime coordinates x^μ with indices $\mu, \nu, \cdots$ and 7-dimensional internal space coordinates y^m with indices $m, n, \cdots$. Denoting the 11-dimensional metric as g_{MN} with the convention $(-,+,\cdots,+)$ and the antisymmetric tensor fields as $F_{MNPQ} = 4\,\partial_{[M}A_{NPQ]}$, the bosonic Einstein-Maxwell field equations are characterized by [20]

$$\begin{aligned} R_M{}^N &= \frac{1}{3}\,F_{MPQR}F^{NPQR} - \frac{1}{36}\delta_M^N\,F_{PQRS}F^{PQRS}, \\ \nabla_M F^{MNPQ} &= -\frac{1}{576}\,E\,\epsilon^{NPQRSTUVWXY}F_{RSTU}F_{VWXY}, \end{aligned} \tag{2.1}$$

where the covariant derivative ∇_M on F^{MNPQ} in the second equation of (2.1) is given by $E^{-1}\partial_M(EF^{MNPQ})$ together with elfbein determinant $E \equiv \sqrt{-g_{11}}$. The 11-dimensional epsilon tensor $\epsilon_{NPQRSTUVWXY}$ with lower indices is purely numerical. The 11-dimensional geometry is a warped product of AdS_4 and the 7-dimensional ellipsoid. We refer the reader to [5, 21] for a derivation of the formula in this section.

The gauged $\mathcal{N}=8$ supergravity theory has self-interaction of a single massless $\mathcal{N}=8$ supermultiplet of spins $\left(2, \frac{3}{2}, 1, \frac{1}{2}, 0^+, 0^-\right)$ with local SO(8) and local SU(8) invariance. There exists a non-trivial effective potential for the scalars that is proportional to the square of the SO(8) gauge coupling g. The 70 real, physical scalars characterized by $(0^+, 0^-)$ of $\mathcal{N}=8$ supergravity parametrize the coset space $E_{7(7)}/\mathrm{SU}(8)$ and they are described by an element $\mathcal{V}(x)$ of the fundamental 56-dimensional representation of $E_{7(7)}$. Any ground state leaving the symmetry unbroken is necessarily AdS_4 space with a cosmological constant

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proportional to g^2. Turning on the scalar fields proportional to the self-dual tensor C_+^{IJKL} of SO(8) yields an $SO(7)^+$-invariant vacuum while turning on pseudo-scalar fields proportional to the anti-self-dual tensor C_-^{IJKL} of SO(8) yields $SO(7)^-$-invariant vacuum. Both $SO(7)^\pm$ vacua are nonsupersymmetric. However, simultaneously turning on both scalar and pseudo-scalar fields proportional to C_+^{IJKL} and C_-^{IJKL}, respectively, one obtains G_2-invariant vacuum with $\mathcal{N} = 1$ supersymmetry [22]. The most general vev of 56-bein preserving G_2-invariance can be parametrized by

$$\phi_{IJKL} = \frac{\lambda}{2\sqrt{2}} \left(\cos\alpha \;\; C_+^{IJKL} + i \sin\alpha \;\; C_-^{IJKL} \right). \tag{2.2}$$

The two vevs (λ, α) in (2.2) are given by functions of the AdS_4 radial coordinate $r \equiv x^4$. The metric formula of [22] generates the 7-dimensional metric from the two input data of AdS_4 vevs (λ, α). The G_2-invariant RG flow is a trajectory in (λ, α)-plane and is parametrized by the AdS_4 radial coordinate r. Instead of using (λ, α), it is convenient to use (a, b) defined by [5]

$$\begin{aligned} a &\equiv \cosh\left(\frac{\lambda}{\sqrt{2}}\right) + \cos\alpha \, \sinh\left(\frac{\lambda}{\sqrt{2}}\right), \\ b &\equiv \cosh\left(\frac{\lambda}{\sqrt{2}}\right) - \cos\alpha \, \sinh\left(\frac{\lambda}{\sqrt{2}}\right). \end{aligned} \tag{2.3}$$

Let us introduce the standard metric of a 7-dimensional ellipsoid. Using the diagonal matrix Q_{AB} given by [5, 22]

$$Q_{AB} = \mathrm{diag}\left(1, 1, 1, 1, 1, 1, 1, \frac{a^2}{b^2}\right), \tag{2.4}$$

the 7-dimensional ellipsoidal metric with the eccentricity $\sqrt{1 - \frac{b^2}{a^2}}$ can be written as

$$ds^2_{EL(7)} = dX^A Q^{-1}_{AB} \, dX^B - \frac{b^2}{\xi^2} \left(X^A \delta_{AB} \, dX^B \right)^2, \tag{2.5}$$

where the $\mathbf{R}^8$ coordinates $X^A (A = 1, \ldots, 8)$ are constrained on the unit round $\mathbf{S}^7$, that is, $\sum_{A=1}^8 (X^A)^2 = 1$, and $\xi^2 \equiv b^2 \, X^A Q_{AB} X^B$ with (2.4) is a quadratic form on the 7-dimensional ellipsoid. The standard metric (2.5) can be rewritten, using the explicit realization between X^A and y^m, in terms of the 7-dimensional coordinates y^m such that

$$ds^2_{EL(7)} = \frac{\xi^2}{a^2} \, d\theta^2 + \sin^2\theta \, d\Omega_6^2, \tag{2.6}$$

where $\theta \equiv y^7$ is the fifth coordinate in 11-dimensions and the quadratic form ξ^2 is given by

$$\xi^2 = a^2 \cos^2\theta + b^2 \sin^2\theta, \tag{2.7}$$

which turns to be 1 for the round $\mathbf{S}^7$ with $(a, b) = (1, 1)$ which has SO(8) symmetry. For other values of a and b, the SO(8) symmetry group is broken down generically to G_2. The metric on the round $\mathbf{S}^6 \simeq \frac{G_2}{\mathrm{SU}(3)}$ is denoted by $d\Omega_6^2$. The geometric parameters (a, b) for the

7-dimensional ellipsoid can be identified with the two vevs (a, b) defined in (2.3). This is one of the reasons why we have introduced (a, b) in (2.3) rather than the original vevs (λ, α).

Applying the Killing vector together with the 28-beins to the metric formula [22], one obtains the inverse metric g^{mn} including the warp factor Δ not yet determined. Substitution of this inverse metric into the definition of warp factor Δ [22] provides a self-consistent equation for Δ. For the G_2-invariant RG flow, solving this equation yields the warp factor

$$\Delta = a^{-1}\,\xi^{-\frac{4}{3}}, \tag{2.8}$$

where ξ is given by (2.7). Then we substitute this warp factor into the inverse metric to obtain the 7-dimensional warped ellipsoidal metric as follows [5, 21]:

$$ds_7^2 = g_{mn}(y)\,dy^m dy^n = \sqrt{\Delta\,a}\,L^2\left(\frac{\xi^2}{a^2}\,d\theta^2 + \sin^2\theta\,d\Omega_6^2\right) = \sqrt{\Delta\,a}\,L^2\,ds^2_{EL(7)}, \tag{2.9}$$

where one can see that the standard 7-dimensional ellipsoidal metric (2.6) is warped by a factor $\sqrt{\Delta\,a}$. The nonlinear metric ansatz finally combines the 7-dimensional metric (2.9) with the four dimensional metric with warp factor to yield the 11-dimensional warped metric with (2.8), (2.7) and (2.9) that solves (2.1) with the appropriate 4-form field strengths below:

$$ds_{11}^2 = \Delta^{-1}\left(dr^2 + e^{2A(r)}\eta_{\mu\nu}dx^\mu dx^\nu\right) + ds_7^2, \tag{2.10}$$

where $r \equiv x^4$ and $\mu, \nu = 1, 2, 3$ with $\eta_{\mu\nu} = \mathrm{diag}(-, +, +)$. The (a, b) are set to

$$a = \sqrt{\frac{6\sqrt{3}}{5}}, \qquad b = \sqrt{\frac{2\sqrt{3}}{5}}, \tag{2.11}$$

for G_2-invariant IR critical point.

The 3-form gauge field with 3-dimensional M2-brane indices may be defined by [7]

$$A_{\mu\nu\rho} = -e^{3A(r)}\,\widetilde{W}(r,\theta)\,\epsilon_{\mu\nu\rho}, \tag{2.12}$$

where $\widetilde{W}(r,\theta)$ is a geometric superpotential [23] to be determined. The θ-dependence of $\widetilde{W}(r,\theta)$ was essential to achieve the M-theory lift of the RG flow. As performed in [24], the G_2-covariant tensors living on the round $\mathbf{S}^6$ can be obtained by using the imaginary octonion basis of $\mathbf{S}^6$. Thus we arrive at the most general G_2-invariant ansatz [5]:

$$A_{4mn} = g(r,\theta)\,F_{mn}, \quad A_{5mn} = h(r,\theta)\,F_{mn}, \quad A_{mnp} = h_1(r,\theta)\,T_{mnp} + h_2(r,\theta)\,S_{mnp}, \tag{2.13}$$

where m, n, p are the $\mathbf{S}^6$ indices and run from 6 to 11. The almost complex structure on the $\mathbf{S}^6$ is denoted by F_{mn} which obeys $F_{mn}F^{nl} = -\delta_m^l$. The S_{mnp} is the parallelizing torsion of the unit round $\mathbf{S}^7$ projected onto the $\mathbf{S}^6$, while the T_{mnp} denotes the 6-dimensional Hodge dual of S_{mnp}. We refer to [24] for further details about these tensors. The above ansatz for gauge field is the most general one which preserves the G_2-invariance and is consistent with the 11-dimensional metric (2.10).

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Through the definition $F_{MNPQ} \equiv 4\,\partial_{[M}A_{NPQ]}$, the ansatz (2.12) generates the field strengths

$$F_{\mu\nu\rho 4} = e^{3A(r)}\,W_r(r,\theta)\,\epsilon_{\mu\nu\rho}, \qquad F_{\mu\nu\rho 5} = e^{3A(r)}\,W_\theta(r,\theta)\,\epsilon_{\mu\nu\rho}, \tag{2.14}$$

while the ansatz (2.13) provides

$$\begin{aligned} F_{mnpq} &= 2\,h_2(r,\theta)\,{}^6\epsilon_{mnpqrs}F^{rs}, & F_{5mnp} &= \tilde{h}_1(r,\theta)\,T_{mnp} + \tilde{h}_2(r,\theta)\,S_{mnp}, \\ F_{4mnp} &= \tilde{h}_3(r,\theta)\,T_{mnp} + \tilde{h}_4(r,\theta)\,S_{mnp}, & F_{45mn} &= \tilde{h}_5(r,\theta)\,F_{mn}, \end{aligned} \tag{2.15}$$

where the coefficient functions which depend on both r and θ are given by [5]

$$\begin{aligned} &W_r = e^{-3A}\partial_r\left(e^{3A}\widetilde{W}\right), && W_\theta = e^{-3A}\partial_\theta\left(e^{3A}\widetilde{W}\right), \\ &\tilde{h}_1 = \partial_\theta h_1 - 3h, && \tilde{h}_2 = \partial_\theta h_2, && \tilde{h}_3 = \partial_r h_1 - 3g, \\ &\tilde{h}_4 = \partial_r h_2, && \tilde{h}_5 = \partial_r h - \partial_\theta g. \end{aligned} \tag{2.16}$$

The mixed field strengths $F_{\mu\nu\rho 5}$, F_{4mnp} and F_{45mn} were new. At both SO(8)-invariant UV and G_2-invariant IR critical points, the 4-dimensional spacetime becomes asymptotically AdS_4 and the mixed field strengths should vanish there. In particular, the $F_{\mu\nu\rho 5}$ and F_{45mn} are proportional to W_θ and $\tilde{h}_5$, respectively, so that they must be subject to the non-trivial boundary conditions: $W_\theta = 0$ and $\tilde{h}_5 = 0$, at both UV and IR critical points. It was checked that the F_{4mnp} goes to zero at both critical points without imposing any boundary condition.

Applying the field strength ansatz (2.14), (2.15) and the metric (2.10) to the 11-dimensional Maxwell equation in (2.1), we have obtained

$$\begin{aligned} \tilde{h}_1 &= 2L\,a^{-3}\,\xi^{-2}\,W_r h_2 - \frac{1}{4}\,L^2\,a^{-3}\,\xi^{-2}\,e^{-3A}\partial_r\left(a^2 e^{3A}\sin^2\theta\tilde{h}_5\right), \\ \tilde{h}_3 &= -2L^{-1}\,\xi^{-2}\,W_\theta h_2 + \frac{1}{4}\,\xi^{-2}e^{-3A}\partial_\theta\left(a^2 e^{3A}\sin^2\theta\tilde{h}_5\right). \end{aligned} \tag{2.17}$$

The 11-dimensional Einstein-Maxwell equations were checked from the warped metric (2.10) and the field strength ansatz (2.14) and (2.15). The 11-dimensional field equations were closed within the field strengths W_r, W_θ, h_2 and $\tilde{h}_5$ (2.16) although they cannot be solved separately without imposing certain ansatz for them. Solving the ansatz one obtained

$$\begin{aligned} h_2 &= \frac{L^3}{2}\sqrt{b^2\,(ab-1)}\;\xi^{-2}\sin^4\theta, \\ \tilde{h}_5 &= 2L^2\sqrt{\frac{(ab-1)}{(a^2+7b^2)^2 - 112\,(ab-1)}}\;(-4a + a^2 b + 7b^3)\,a^{-\frac{1}{2}}\sin^2\theta, \end{aligned} \tag{2.18}$$

and

$$\begin{aligned} W_r &= -\frac{1}{2L}\,a^2\left[a^5\cos^2\theta + a^2 b\,(ab-2)\,(4+3\cos 2\theta) + b^3\,(7ab-12)\sin^2\theta\right], \\ W_\theta &= -\frac{a^{\frac{3}{2}}\left[48\,(1-ab) + (a^2-b^2)\,(a^2+7b^2)\right]}{\sqrt{(a^2+7b^2)^2 - 112\,(ab-1)}}\,\sin\theta\cos\theta. \end{aligned} \tag{2.19}$$

It turned out that the solutions (2.18) and (2.19) actually consist of an exact solution to the 11-dimensional supergravity, provided that the deformation parameters (a, b) of the 7-ellipsoid and the domain wall amplitude $A(r)$ develop in the AdS_4 radial direction along the G_2-invariant RG flow. Finally, the geometric superpotential in (2.12) yields

$$\widetilde{W} = \frac{a^{\frac{3}{2}}\left(\left[48\,(1-ab) + (a^2-b^2)\,(a^2+7b^2)\right]\cos^2\theta + 8\,(1-ab) + b^2\,(a^2+7b^2)\right)}{2\sqrt{(a^2+7b^2)^2 - 112\,(ab-1)}}. \tag{2.20}$$

Therefore, all the field strengths in (2.14) and (2.15) are determined via (2.18), (2.19) and (2.17). The 3-form gauge field (2.12) is also determined via (2.20).

What is the dual gauge theory? Let us recall the U(2) × U(2) Chern-Simons matter theory where the matter fields consist of seven flavors $\Phi_i (i = 1, 2, \cdots, 7)$ transforming in the adjoint with flavor symmetry G_2. There exist standard Chern-Simons terms with levels for the gauge groups $(k, -k)$ with $k = 1, 2$. The Φ_i forms a septet **7** of the $\mathcal{N} = 1$ theory. When we turn on the mass perturbation in the gauged supergravity, the dual theory flows from the UV to the IR. In the dual field theory, one integrates out the massive field Φ_8(which is a singlet **1** of G_2 with adjoint index) characterized by the superpotential $\frac{1}{2}m\Phi_8^2$, at a low enough scale, and then this results in the sixth order superpotential [12].

3 KK spectrum of minimally coupled scalar

Let us assume the system that a minimally coupled scalar field is interacting with the gravitational field. The action for a minimally coupled scalar field in the background which is a warped product of AdS_4 and 7-dimensional ellipsoid is given by [11]

$$S = \int d^{11}x\sqrt{-g}\left[-\frac{1}{2}(\partial\phi)^2\right]. \tag{3.1}$$

The equation of motion from this action (3.1) is given by

$$\Box\phi = 0, \tag{3.2}$$

where $\Box$ is the 11-dimensional Laplacian. Using the separation of variables

$$\phi = \Phi(x^\mu, r)Y(y^m), \tag{3.3}$$

and substituting (3.3) into (3.2), one writes (3.2) as

$$Y(y^m)\Box_4\Phi(x^\mu, r) + \Phi(x^\mu, r)\mathcal{L}Y(y^m) = 0, \tag{3.4}$$

where $\Box_4$ denotes the AdS_4 Laplacian and $\mathcal{L}$ denotes a differential operator acting on 7-dimensional ellipsoid and is given by

$$\mathcal{L} \equiv \frac{\Delta^{-1}}{\sqrt{-g_{11}}}\,\partial_M\left(\sqrt{-g_{11}}\,g_{11}^{MN}\,\partial_N\right) = \frac{\Delta^{-\frac{3}{4}}}{\sqrt{g_7}}\,\partial_m\left(a^{-\frac{1}{2}}\,L^{-2}\,\Delta^{-\frac{3}{4}}\,\sqrt{g_7}\,g_7^{mn}\,\partial_n\right), \tag{3.5}$$

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where g^7_{mn} and g^{11}_{MN} are given by the metrics (2.5) and (2.10) respectively. In particular, the 7-dimensional metric is given by

$$g^7_{mn}=\begin{pmatrix} \frac{1}{3}(2+c_{2\theta}) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & s^2_\theta s^2_{\theta_6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4}s^2_\theta s^2_{\theta_1} s^2_{\theta_6} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4}s^2_\theta s^2_{\theta_1} s^2_{\theta_6} & \frac{1}{4}c_{\alpha_1} s^2_\theta s^2_{\theta_1} s^2_{\theta_6} & \frac{1}{2}c_{\alpha_1} s^2_\theta s^2_{\theta_1} s^2_{\theta_6} & 0 \\ 0 & 0 & 0 & \frac{1}{4}c_{\alpha_1} s^2_\theta s^2_{\theta_1} s^2_{\theta_6} & \frac{1}{4}s^2_\theta s^2_{\theta_1} s^2_{\theta_6} & \frac{1}{2}s^2_\theta s^2_{\theta_1} s^2_{\theta_6} & 0 \\ 0 & 0 & 0 & \frac{1}{2}c_{\alpha_1} s^2_\theta s^2_{\theta_1} s^2_{\theta_6} & \frac{1}{2}s^2_\theta s^2_{\theta_1} s^2_{\theta_6} & s^2_\theta s^2_{\theta_6} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & s^2_\theta, \end{pmatrix}, \quad (3.6)$$

where we use the simplified notations $c_{2\theta} \equiv \cos 2\theta$ and $s_\theta \equiv \sin\theta$ and so on. Let us introduce the angular coordinates $y^m \equiv (\theta, \theta_1, \alpha_1, \alpha_2, \alpha_3, \theta_5, \theta_6)$ [5] parametrizing the $\mathbf{S}^7$ inside $\mathbf{R}^8$ with $\sum_{A=1}^8 (X^A)^2 = 1$ where the relation to the rectangular coordinates is

$$\begin{aligned} X^1 + iX^2 &= \sin\theta \sin\theta_6 \sin\theta_1 \cos\left(\frac{1}{2}\alpha_1\right) e^{\frac{i}{2}(\alpha_2+\alpha_3)} e^{i\theta_5}, \\ X^3 + iX^4 &= \sin\theta \sin\theta_6 \sin\theta_1 \sin\left(\frac{1}{2}\alpha_1\right) e^{-\frac{i}{2}(\alpha_2-\alpha_3)} e^{i\theta_5}, \\ X^5 + iX^6 &= \sin\theta \sin\theta_6 \cos\theta_1 e^{i\theta_5}, \\ X^7 &= \sin\theta \cos\theta_6, \\ X^8 &= \cos\theta. \end{aligned} \quad (3.7)$$

This is $\mathbf{R}^8$ embedding of $\mathbf{S}^7$ with $\frac{G_2}{\mathrm{SU}(3)}$ base and the relation to the Hopf fibration on $\mathbf{CP}^3$ is given in [5]. The quadratic form (2.7) is given by $\xi^2 = \frac{2}{5}\sqrt{3}(2+c_{2\theta})$ and the warp factor (2.8) is given by $\Delta = \frac{5^{\frac{7}{6}}}{2^{\frac{7}{6}} \times 3^{\frac{13}{12}} (2+c_{2\theta})^{\frac{2}{3}}}$. The equation of ellipsoid is $\frac{5\sum_{A=1}^7 (X^A)^2}{6\sqrt{3}} + \frac{5(X^8)^2}{2\sqrt{3}} = 1$.

Let us find out the eigenfuction $Y(y^m)$ of the differential operator $\mathcal{L}$

$$\mathcal{L} Y(y^m) = -m^2 Y(y^m). \quad (3.8)$$

Then the equation (3.4) leads to the equation of motion of a massive scalar field in AdS_4 as follows:

$$\Box_4 \Phi(x^\mu, r) - m^2 \Phi(x^\mu, r) = 0. \quad (3.9)$$

Therefore, the 11-dimensional minimally coupled scalar provides a tower of KK modes which are all massive scalars (3.9) with masses m^2 determined by the eigenvalues of the above differential operator $\mathcal{L}$.

Let us recall that the G_2 symmetry is the isometry group of the metric. The isometry of round $\mathbf{S}^6$ only is given by SO(7) which contains G_2 as a subgroup and the Killing vector associated to the G_2 symmetry is given by

$$K^a = K_A{}^a \partial^A = \left[X^B (T^a)_{BA} - X^C (T^a)_{AC} \right] \partial^A, \quad (3.10)$$

where $A, B = 1, 2, \cdots, 7$ for rectangular coordinates, $a = 1, 2, \cdots, 14$ for adjoint indices, and T^a are traceless antihermitian matrices and the generators of G_2. The dimension of G_2

is 14. The explicit form of these is given by (A.4) and (A.5) of appendix A. We have checked that the metric g^7_{mn} (3.6) has vanishing Lie derivative [25] with respect to the Killing vector fields K^a. That is, $K^{p\,a}\partial_p g^7_{mn} + (\partial_m K^{p\,a}) g^7_{pn} + (\partial_n K^{p\,a}) g^7_{mp} = 0$ where $K^a = K_q{}^a \partial^q = g_{qp} K^{p\,a} \partial^q$. Note that in this check, we need to work with the angular coordinates using the chain rules between the rectangular coordinates and angular coordinates (3.7): after introducing the radial coordinate and obtaining the Jacobian, this radial coordinate is set to one.

In an irreducible representation of G_2 labeled by the Dynkin labels (λ, μ), the quadratic Casimir operator of G_2

$$\mathcal{C}_2 \equiv \sum_{a=1}^{14} (K^a)^2, \tag{3.11}$$

has eigenvalues [26, 27]

$$\mathcal{C}_2(\lambda, \mu) = -16 \left(\lambda^2 + \frac{1}{3}\mu^2 + \lambda\mu + 3\lambda + \frac{5}{3}\mu \right), \tag{3.12}$$

for (λ, μ) representation. Note that $\sum_{a=1}^{14} (T^a)^2 = -8$. The explicit form for (3.11) is given by (A.2) of appendix A.

The spin-2 massive $\mathcal{N} = 8$ supermultiplet [25] at level n is characterized by the SO(8) Dynkin labels $(n, 0, 0, 0)$, this breaks into the SO(7) Dynkin labels $(0, 0, n)$, and finally the massive multiplets of $\mathcal{N} = 8$ for $n = 1, 2, \cdots$, are decomposed into $(0, 0) \oplus (0, 1) \oplus \cdots \oplus (0, n)$ under the G_2 symmetry. In particular, one has, with the help of [28],

$$\begin{aligned}
\mathrm{SO}(8) &\to \mathrm{SO}(7) \to G_2, \\
\mathbf{8}_v(1,0,0,0) &\to \mathbf{8}(0,0,1) \to \mathbf{1}(0,0) \oplus \mathbf{7}(0,1), \\
\mathbf{35}_v(2,0,0,0) &\to \mathbf{35}(0,0,2) \to \mathbf{1}(0,0) \oplus \mathbf{7}(0,1) \oplus \mathbf{27}(0,2), \\
\mathbf{112}_v(3,0,0,0) &\to \mathbf{112}'(0,0,3) \to \mathbf{1}(0,0) \oplus \mathbf{7}(0,1) \oplus \mathbf{27}(0,2) \oplus \mathbf{77}'(0,3), \\
(n,0,0,0) &\to (0,0,n) \to (0,0) \oplus (0,1) \cdots \oplus (0,n).
\end{aligned} \tag{3.13}$$

Then the relevant eigenvalues for quadratic Casimir from (3.12) when $\lambda = 0$ are given by

$$\mathcal{C}_2(0, \mu) = -\frac{16}{3}\mu\,(\mu + 5)\,. \tag{3.14}$$

Note that the factor $\mu\,(\mu + 5)$ occurs in the quadratic Casimir operator of SO(7).

What are the corresponding eigenfunctions? It is well-known that the scalar spherical harmonic for D-sphere $\mathbf{S}^D$ is characterized by each independent component of totally symmetric traceless tensor of rank n. Namely, they are linear combinations of products of μ factors of X^A's

$$Y_{(0,\mu)}(X^A, A \neq 8) = C^{(0,\mu)}_{i_1 i_2 \cdots i_\mu} X^{i_1} X^{i_2} \cdots X^{i_\mu}, \tag{3.15}$$

where $C^{(0,\mu)}_{i_1 i_2 \cdots i_\mu}$ is a $(0, \mu)$-tensor independent of the X^A's and is symmetric in $i_1 i_2 \cdots i_\mu$. Furthermore, they are traceless in the sense that $C^{(0,\mu)}_{i_1 i_2 \cdots i_\mu} \delta^{i_m i_n} = 0$ for any $1 \leq m, n \leq \mu$ [29].

Since $\mathcal{C}_2$ doesn't act on θ(See also (A.1) or (A.2)), one multiplies the expression (3.15) by any function of θ or X^8. Let us write down the eigenmodes as

$$Y(y^m) = Y_{(0,\mu)}(X^A, A \neq 8)\, H(u), \qquad u \equiv \cos^2\theta. \tag{3.16}$$

In the tensor product of $\mathbf{7}\times\mathbf{7} = [\mathbf{1}\oplus\mathbf{27}]_S \oplus [\mathbf{7}\oplus\mathbf{14}]_A$ between the defining representation $\mathbf{7} = (0,1)$ of G_2, the tensors transforming as the symmetric part are given by [26]

$$\begin{aligned} Y_{\mathbf{1}(0,0)AB} &= \frac{1}{7}\delta_{AB}\delta_{CD}V^C W^D, \\ Y_{\mathbf{27}(0,2)AB} &= \left[\frac{1}{2}\left(\delta_{AC}\delta_{BD}+\delta_{AD}\delta_{BC}\right) - \frac{1}{7}\delta_{AB}\delta_{CD}\right] V^C W^D, \end{aligned} \tag{3.17}$$

where V^A and W^B transform as $\mathbf{7}$ of G_2 and have nothing to do with X^A. Then one sees that $Y_{\mathbf{27}(0,2)AB}$ plays the role of $C^{(0,2)}_{AB}$ in (3.15) because $Y_{\mathbf{27}(0,2)AB}$ is symmetric under the indices A and B and traceless due to the fact that $\sum_{A=1}^{7} Y_{\mathbf{27}(0,2)AA} = 0$ from (3.17).[1]

Now let us solve the differential equation (3.8) with (3.16). The differential operator (3.5) can be computed explicitly and it is given in the appendix A (A.1) in terms of the angular variables. Also this differential operator can be expressed in terms of the rectangular coordinates via (A.2) and (A.3). Since the quadratic Casimir operator has explicit eigenvalues of (3.14), it is obvious that the eigenvalue equation (3.8) can be solved more easily by using the rectangular coordinates rather than the angular coordinates. When $\mathcal{C}_2$ acts on $Y(y^m)$ in (3.16), the only nonzero contributions arise if it acts on $Y_{(0,\mu)}(X^A)$ because $\mathcal{C}_2$ doesn't act on u as before.

On the other hand, the remaining other piece (A.3) of $\mathcal{L}$ consisting of the second derivative and first derivative with respect to θ or u can act on either $Y_{(0,\mu)}(X^A)$ or $H(u)$. The former arises because the $X^A(A = 1, 2, \cdots, 7)$ depends on the variable θ from (3.7). Since $Y_{(0,\mu)}(X^A)$ is written in terms of rectangular coordinates, it is convenient to rewrite the remaining operator in terms of rectangular coordinates also using the chain rules, as in (A.3). One can compute the action of this remaining operator on $Y_{(0,\mu)}(X^A)$ explicitly. Moreover when the remaining operator acts on the function $H(u)$, then one obtains the second and first derivatives of $H(u)$ with respect to u. In this way, one performs these computations for $\mu = 0, 1, 2, 3$ and expects the final expression for general μ. Actually we have checked this for $\mu = 4$. It turns out that the eigenvalue problem leads to the following differential equation

$$(1-u)uH'' + [c - (a_+ + a_- + 1)u]\, H' - a_+ a_- H = 0, \tag{3.18}$$

[1] Although the full solution preserves the G_2 symmetry, the metric preserves a bigger SO(7) symmetry from (2.6). Since the KK modes see only the metric background, one can take the different approach by using the fact that the Laplacian eigenvalue problem on $\mathbf{S}^6$ is known. The Killing vector associated to the SO(7) symmetry can be obtained from (3.10) by taking 21 generators of SO(7), that are 7×7 matrices, as $(T_{ij})_{pq} = \frac{1}{2}(\delta_{ip}\delta_{jq} - \delta_{iq}\delta_{jp})$ with the property $\sum_{i,j=1}^{7}(T_{ij})^2 = -\frac{3}{2}$. Then one can check that the quadratic Casimir operator of SO(7) corresponding to (3.11) is identical to the expressions of inside bracket in (A.2). Also the eigenfunctions (3.15) can be factorized by $\sin^\mu\theta\, Y_\mu$ where Y_μ is so-called SO(7) spherical harmonics which depend on $(\theta_1, \alpha_1, \alpha_2, \alpha_3, \theta_5, \theta_6)$. When we act the above quadratic Casimir of SO(7) on this spherical harmonics Y_μ, we get $-\mu(\mu+5)Y_\mu$, as usual. We thank the referee for raising this point.

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where the primes denote the derivatives with respect to u and we introduce the following quantities

$$a_{\pm}\equiv\frac{1}{6}\left(9+3\mu\pm\sqrt{6\mu^2+30\mu+81+\frac{72}{5}m^2\hat{L}^2}\right),\qquad c=\frac{1}{2},\qquad \frac{2\sqrt{\frac{36\sqrt{2}3^{\frac{1}{4}}}{25\sqrt{5}}}}{L}\equiv\frac{1}{\hat{L}}. \tag{3.19}$$

The complicated quantity $\sqrt{\frac{36\sqrt{2}3^{\frac{1}{4}}}{25\sqrt{5}}}$ is the superpotential at the IR critical point and from the domain wall equation, the scale factor $A(r)$ behaves as $A(r)\sim\frac{2\sqrt{\frac{36\sqrt{2}3^{\frac{1}{4}}}{25\sqrt{5}}}}{L}r$ at the IR end of the flow [12] and we introduce a new quantity $\hat{L}$ above in the spirit of [11]. Note that the contribution from $\mathcal{C}_2$ occurs only in the term of H in (3.18) while the contributions from the remaining operator occur all the terms in H'', H' or H of (3.18). We have also checked that the eigenvalue equation (3.8) holds (3.18) when $\mathcal{L}$ and $Y(y^m)$ are written in terms of angular coordinates via (A.1), (3.15), (3.16) and (3.7): tracelessness of $(0,\mu)$ tensor is crucial.

The solution for (3.18) which is regular at $u=0$ is given by

$$H(u)={}_2F_1\left(a_-,a_+;c;u\right), \tag{3.20}$$

which is convergent for $|u|<1$ for arbitrary a_-,a_+ and c. When $a_-=-j$ for nonnegative integer j, this hypergeometric function becomes a polynomial in u of order j. Then the KK spectrum of minimally coupled scalar can be obtained by putting $a_-=-j$ in (3.19) and solving for m^2. Then the mass-squared in AdS_4 can be written, in terms of μ and j, as

$$m^2=\frac{5}{24\hat{L}^2}\left(12j^2+12j\mu+36j+\mu^2+8\mu\right). \tag{3.21}$$

Plugging (3.20) into (3.16) together with (3.19), one obtains the full eigenfunctions

$$Y(y^m)=C^{(0,\mu)}_{i_1i_2\cdots i_\mu}\left(\prod_{k=1}^{\mu}X^{i_k}\right){}_2F_1\left(-j,3+\mu+j;\frac{1}{2};(X^8)^2\right). \tag{3.22}$$

In this case, the hypergeometric functions are polynomials and let us write down few cases according to the KK excitation number j

$$\begin{aligned}
j=0:&\quad Y(y^m)\sim Y_{(0,\mu)}(X^A),\\
j=1:&\quad Y(y^m)\sim Y_{(0,\mu)}(X^A)\left[1-2(4+\mu)(X^8)^2\right],\\
j=2:&\quad Y(y^m)\sim Y_{(0,\mu)}(X^A)\left[1-4(5+\mu)(X^8)^2+\frac{4}{3}(5+\mu)(6+\mu)(X^8)^4\right],\\
j=3:&\quad Y(y^m)\sim Y_{(0,\mu)}(X^A)\left[1-6(6+\mu)(X^8)^2+4(6+\mu)(7+\mu)(X^8)^4\right.\\
&\qquad\qquad\left.-\frac{8}{15}(6+\mu)(7+\mu)(8+\mu)(X^8)^6\right],
\end{aligned} \tag{3.23}$$

where $A\neq 8$. The appearance of hypergeometric function in the eigenfunction for the 7-dimensional Laplacian operator is not so special and one sees the similar feature in the compactification of $Q^{1,1,1}$ space [30].

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The dimension of the CFT operators dual to the KK modes (3.22) can be determined from the usual AdS/CFT correspondence [2]

$$\Delta(\Delta-3)=m^2\hat{L}^2. \tag{3.24}$$

The $OSp(1|4)$ supermultiplets with spin-2 components are massless graviton multiplet($s_0=\frac{3}{2}$) with $D(3,2)$ denoted by Class 3 of [18] and massive graviton multiplet($s_0=\frac{3}{2}$) with $D(E_0+\frac{1}{2},2)$ where $E_0>\frac{5}{2}$ denoted by Class 4 of [18]. As recognized in [12], the massless graviton multiplet has conformal dimension $\Delta=3$(the ground state component has dimension $\Delta_0=\frac{5}{2}$ and see the table 5 of [12]). This $\mathcal{N}=1$ massless graviton multiplet characterized by $SD\left(\frac{5}{2},\frac{3}{2}|1\right)$ originates from the $\mathcal{N}=2$ massless graviton multiplet characterized by $SD\left(2,1,0|2\right)$ where the element zero stands for $\mathrm{U}(1)_R$ charge [31]. Similarly $\mathcal{N}=1$ massive graviton multiplet $SD\left(E_0+\frac{1}{2},\frac{3}{2}|\,1\right)$ with $E_0\geq 2$ originates from the $\mathcal{N}=2$ massive long graviton multiplet or massive short graviton multiplet.

Based on the findings (3.22), one can study the boundary operators dual to the KK modes by (3.22). The theory has matter multiplet in seven flavors $\Phi^1,\Phi^2,\cdots,\Phi^7$ transforming in the adjoint with flavor symmetry under which the matter multiplet forms a **7** of G_2 of the $\mathcal{N}=1$ theory [12]. The Φ^8 is a singlet **1** of G_2. The gauge theory conjectured to be dual to the G_2 $\mathcal{N}=1$ supergravity background in this paper is a deformation of ABJM theory [1] by a superpotential term quadratic in Φ^8. The gauge theory also has G_2 symmetry where G_2 symmetry corresponds to the global rotations of $\Phi^1,\Phi^2,\cdots,\Phi^7$ into one another. One identifies the Φ^A fields where $A\neq 8$ with the coordinates $X^A(A\neq 8)$ and Φ^8 with the coordinate X^8 up to normalization. One can read off the dual operators corresponding to each of the KK modes.

In table 1,[2] we present a few of these modes and also provide the structure of the dual gauge theory operators from (3.22). The branching rule for SO(8) into G_2 is given by (3.13). The quantum number μ of G_2 is characterized by the Dynkin label $(0,\mu)$ as before. The KK excitation mode j is nonnegative integer and this makes the hypergeometric funtion be finite. The conformal dimension of dual SCFT operator is given by (3.24). Once the mass-squared formula is used via (3.21), then this conformal dimension is fixed. Starting with the $\mathcal{N}=1$ SCFT operator denoted by $\Phi_{\alpha\beta\gamma}$ corresponding to the massless graviton multiplet, one can construct a tower of KK modes by multiplying $\Phi^A(A\neq 8)$ for $j=0$ modes. In general, one expects that for general quantum number μ of G_2, the operator is given by the product of $\Phi_{\alpha\beta\gamma}$ with μ factors of $\Phi^A(A\neq 8)$ where the μ indices are symmetrized. For nonzero j's, the explicit form (3.23) of hypergeometric functions is useful to identify the corresponding $\mathcal{N}=1$ SCFT operators. For general j, there exists a polynomial up to the order $2j$ in Φ^8 multiplied by $\Phi_{\alpha\beta\gamma}$.

According to the observation of [14], there exists a supersymmetric RG flow from $\mathcal{N}=1$ G_2-invariant fixed point to $\mathcal{N}=2$ $\mathrm{SU}(3)\times\mathrm{U}(1)_R$-invariant fixed point. One describes the

[2] In these three-dimensional SCFTs, there exist monopole operators [10] available for $k=1,2$ that should be used to obtain the correct gauge invariant operators. In the same spirit of [11], we present only the "schematic" expressions of the dual gauge theory operators which do not contain these monopole operators. The corresponding $\mathcal{N}=1$ SCFT operators in Chern-Simons matter theory hold for the gauge group $\mathrm{U}(2)\times\mathrm{U}(2)$ with $k=1,2$.

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SO(8)	G_2	j	Δ	$m^2\hat{L}^2$	$\mathcal{N}=1$ SCFT Operator
$\mathbf{1}(0,0,0,0)$	$\mathbf{1}(0,0)$	0	3	0	$\Phi_{\alpha\beta\gamma}$
$\mathbf{8}_v(1,0,0,0)$	$\mathbf{1}(0,0)$	1	5	10	$\Phi_{\alpha\beta\gamma}\left[1-8(\Phi^8)^2\right]$
	$\mathbf{7}(0,1)$	0	$\frac{1}{4}(6+\sqrt{66})$	$\frac{15}{8}$	$\Phi_{\alpha\beta\gamma}\Phi^A$
$\mathbf{35}_v(2,0,0,0)$	$\mathbf{1}(0,0)$	2	$\frac{1}{2}(3+\sqrt{109})$	25	$\Phi_{\alpha\beta\gamma}\left[1-20(\Phi^8)^2+40(X^8)^4\right]$
	$\mathbf{7}(0,1)$	1	$\frac{1}{4}(6+\sqrt{266})$	$\frac{115}{8}$	$\Phi_{\alpha\beta\gamma}\Phi^A\left[1-10(\Phi^8)^2\right]$
	$\mathbf{27}(0,2)$	0	$\frac{1}{6}(9+\sqrt{231})$	$\frac{25}{6}$	$\Phi_{\alpha\beta\gamma}\Phi^{(A}\Phi^{B)}$
$\mathbf{112}_v(3,0,0,0)$	$\mathbf{1}(0,0)$	3	$\frac{3}{2}(1+\sqrt{21})$	45	$\Phi_{\alpha\beta\gamma}\left[1-36(\Phi^8)^2+168(\Phi^8)^4-\frac{896}{5}(\Phi^8)^6\right]$
	$\mathbf{7}(0,1)$	2	$\frac{1}{4}(6+\sqrt{546})$	$\frac{255}{8}$	$\Phi_{\alpha\beta\gamma}\Phi^A\left[1-24(\Phi^8)^2+56(\Phi^8)^4\right]$
	$\mathbf{27}(0,2)$	1	$\frac{1}{6}(9+\sqrt{771})$	$\frac{115}{6}$	$\Phi_{\alpha\beta\gamma}\Phi^{(A}\Phi^{B)}\left[1-12(\Phi^8)^2\right]$
	$\mathbf{77}'(0,3)$	0	$\frac{1}{4}(6+\sqrt{146})$	$\frac{55}{8}$	$\Phi_{\alpha\beta\gamma}\Phi^{(A}\Phi^B\Phi^{C)}$

Table 1. The first few spin-2 components of the massive(and massless) graviton multiplets. For each multiplet we present $\mathrm{SO}(8), G_2$ Dynkin labels (3.13), the KK excitation number j, the dimension Δ (3.24) of the spin-2 component of the multiplet, the mass-squared $m^2\hat{L}^2$ (3.21) of the AdS_4 field and the corresponding dual SCFT operator.

branching rules [28] of G_2 into SU(3) as follows:

$$\begin{aligned}
G_2 &\to \mathrm{SU}(3),\\
\mathbf{1}(0,0) &\to \mathbf{1}(0,0),\\
\mathbf{7}(0,1) &\to \mathbf{3}(1,0)\oplus\overline{\mathbf{3}}(0,1)\oplus\mathbf{1}(0,0),\\
\mathbf{27}(0,2) &\to \mathbf{6}(2,0)\oplus\mathbf{8}(1,1)\oplus\overline{\mathbf{6}}(0,2)\oplus\mathbf{3}(1,0)\oplus\overline{\mathbf{3}}(0,1)\oplus\mathbf{1}(0,0),\\
\mathbf{77}'(0,3) &\to \mathbf{10}(3,0)\oplus\overline{\mathbf{15}}(1,2)\oplus\overline{\mathbf{10}}(0,3)\oplus\mathbf{15}(2,1)\oplus\mathbf{6}(2,0)\oplus\mathbf{8}(1,1)\oplus\overline{\mathbf{6}}(0,2)\\
&\quad\oplus\mathbf{3}(1,0)\oplus\overline{\mathbf{3}}(0,1)\oplus\mathbf{1}(0,0),\\
(0,\mu) &\to [(\mu,0)\oplus(\mu-1,1)\oplus\cdots(0,\mu)]\oplus[(\mu-1,0)\oplus(\mu-2,1)\oplus\cdots(0,\mu-1)]\\
&\quad\oplus\cdots\oplus[(1,0)\oplus(0,1)]\oplus(0,0),
\end{aligned}\tag{3.25}$$

where the dimension [26] of G_2 representation $(0,\mu)$ is given by $\frac{1}{120}\prod_{i=1}^4(\mu+\mathbf{i})(\mathbf{2}\mu+\mathbf{5})$. At the level of $\mu=0$ excitation, this is the familiar massless graviton multiplet in AdS_4 and corresponds to the stress energy tensor in the SCFT. In the standard $\mathcal{N}=2$ stress energy superfield $\mathcal{T}^{(0)}_{\alpha\beta}$ in the notation of [11], the components are given by a vector boson that is related to $\mathrm{U}(1)_R$ symmetry, two fermionic supersymmetry generators, and energy momentum tensor. Then we denote $D_\alpha\mathcal{T}^{(0)}_{\beta\gamma}$ corresponding to the $\mathcal{N}=1$ massless graviton multiplet by $\Phi_{\alpha\beta\gamma}$ which has $\mathcal{N}=1$ supercurrent and the energy momentum tensor in components [12]. Here D_α is an $\mathcal{N}=1$ superderivative. As explained before, the massless graviton multiplet$\left(s_0=\frac{3}{2}\text{ in }\mathcal{N}=1\text{ notation}\right)$ with $D(3,2)$ is classified by the Class 3 of [18] and has conformal dimension $\Delta=3\left(\text{the ground state component has dimension }\Delta_0=\frac{5}{2}\right)$.

Now let us describe the massive graviton multiplet. At the level of $\mu=1$ excitation, the $\mathbf{7}$ representation of G_2 breaks into three different representations of SU(3) as above. In $\mathcal{N}=2$ theory, $\mathbf{3}$ representation of SU(3) corresponds to $\mathcal{Z}^A$ where $A=1,2,3$ and its complex conjugates $\overline{\mathbf{3}}$ representation of SU(3) corresponds to $\overline{\mathcal{Z}}_A$ [8–11]. Then the $\mathcal{N}=1$ SCFT operator goes to the $\mathcal{N}=2$ SCFT operators [11] schematically as follows:

$$\Phi_{\alpha\beta\gamma}\Phi^A\to\mathcal{T}^{(0)}_{\alpha\beta}\left[\mathcal{Z}^A\oplus\overline{\mathcal{Z}}_A\oplus(\mathcal{Z}^4+\overline{\mathcal{Z}}_4)\right],\tag{3.26}$$

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where $\mathcal{Z}^4$ and $\overline{\mathcal{Z}}_4$ are singlets of SU(3) and the sum of these corresponds to the $\mathcal{N}=1$ superfield Φ^7. According to the observation of [11], the last representation of (3.26) belongs to the $\mathcal{N}=2$ short graviton multiplet. The other representations of (3.26) belong to the $\mathcal{N}=2$ long graviton multiplet. For $j=1$ case in the singlet **1** of G_2, the structure of $\mathcal{N}=1$ SCFT operator can be obtained from the expression of hypergeometric function for $j=1$ case (3.23).

At the level of $\mu=2$ excitation, the **27** representation of G_2 breaks into six different representations of SU(3). Then the $\mathcal{N}=1$ SCFT operator goes to the $\mathcal{N}=2$ SCFT operators [11] with the same order for SU(3) representations (3.25)

$$\Phi_{\alpha\beta\gamma}\Phi^{(A}\Phi^{B)} \to \mathcal{T}^{(0)}_{\alpha\beta}\left[\mathcal{Z}^{(A}\mathcal{Z}^{B)} \oplus \left(\mathcal{Z}^A\overline{\mathcal{Z}}_B - \frac{1}{3}\delta^A_B\mathcal{Z}^C\overline{\mathcal{Z}}_C\right) \oplus \overline{\mathcal{Z}}_{(A}\overline{\mathcal{Z}}_{B)} \right.$$
$$\left. \oplus\, \mathcal{Z}^A(\mathcal{Z}^4+\overline{\mathcal{Z}}_4) \oplus \overline{\mathcal{Z}}_A(\mathcal{Z}^4+\overline{\mathcal{Z}}_4) \oplus (\mathcal{Z}^4+\overline{\mathcal{Z}}_4)^2\right]. \qquad (3.27)$$

The right hand side represents each SU(3) representation, term by term, in (3.25) exactly. The second representation can be obtained from the tensor product of **3** and $\overline{\mathbf{3}}$ which leads to **8** and corresponds to the massless vector multiplet. On the other hand, the $\mathcal{N}=1$ massless vector multiplet has conformal dimension $\Delta=2$ for spin-1(the ground state component has dimension $\Delta_0=\frac{3}{2}$ and see the table 5 of [12]). This $\mathcal{N}=1$ massless vector multiplet characterized by $SD\left(\frac{3}{2},\frac{1}{2}\middle|1\right)$ originates from the $\mathcal{N}=2$ massless vector multiplet characterized by $SD(1,0,0|2)$ [31]. The last three representations of (3.27) are obtained from (3.26) by multiplying $\left(\mathcal{Z}^4+\overline{\mathcal{Z}}_4\right)$. The last representation of (3.27) belongs to the $\mathcal{N}=2$ short graviton multiplet [11]. The first three representations can be obtained from the fact that the product $\Phi^{(A}\Phi^{B)}$ has three possible cases: two products of $\mathcal{Z}^A$'s, two products $\overline{\mathcal{Z}}_A$'s and the product of $\mathcal{Z}^A$ and $\overline{\mathcal{Z}}_A$. For $j=1,2$ cases in the $\mathbf{7},\mathbf{1}$ of G_2, the structure of $\mathcal{N}=1$ SCFT operator is read off from the expression of hypergeometric function for $j=1,2$ cases (3.23).

Finally, at the level of $\mu=3$ excitation, the $\mathbf{77}'$ representation of G_2 breaks into ten different representations of SU(3). Then the $\mathcal{N}=1$ SCFT operator goes to the $\mathcal{N}=2$ SCFT operators [11]

$$\Phi_{\alpha\beta\gamma}\Phi^{(A}\Phi^B\Phi^{C)} \to \mathcal{T}^{(0)}_{\alpha\beta}\left[\mathcal{Z}^{(A}\mathcal{Z}^B\mathcal{Z}^{C)} \oplus \left(\overline{\mathcal{Z}}_{(A}\overline{\mathcal{Z}}_{B)}\mathcal{Z}^C - \frac{1}{3}\delta^C_{(A}\overline{\mathcal{Z}}_{B)}\overline{\mathcal{Z}}_D\mathcal{Z}^D\right) \oplus \overline{\mathcal{Z}}_{(A}\overline{\mathcal{Z}}_B\overline{\mathcal{Z}}_{C)}\right.$$
$$\oplus \left(\mathcal{Z}^{(A}\mathcal{Z}^{B)}\overline{\mathcal{Z}}_C - \frac{1}{3}\delta^{(A}_C\mathcal{Z}^{B)}\mathcal{Z}^D\overline{\mathcal{Z}}_D\right) \oplus \mathcal{Z}^{(A}\mathcal{Z}^{B)}(\mathcal{Z}^4+\overline{\mathcal{Z}}_4)$$
$$\oplus \left(\mathcal{Z}^A\overline{\mathcal{Z}}_B - \frac{1}{3}\delta^A_B\mathcal{Z}^C\overline{\mathcal{Z}}_C\right)(\mathcal{Z}^4+\overline{\mathcal{Z}}_4) \oplus \overline{\mathcal{Z}}_{(A}\overline{\mathcal{Z}}_{B)}(\mathcal{Z}^4+\overline{\mathcal{Z}}_4)$$
$$\left.\oplus\, \mathcal{Z}^A(\mathcal{Z}^4+\overline{\mathcal{Z}}_4)^2 \oplus \overline{\mathcal{Z}}_A(\mathcal{Z}^4+\overline{\mathcal{Z}}_4)^2 \oplus (\mathcal{Z}^4+\overline{\mathcal{Z}}_4)^3\right]. \qquad (3.28)$$

The second representation in the right hand side can be obtained from the tensor product of **8** and $\overline{\mathbf{3}}$ which leads to $\overline{\mathbf{15}}$. The first four representations can be obtained from the fact that the product $\Phi^{(A}\Phi^B\Phi^{C)}$ has four possible cases: three products of $\mathcal{Z}^A$'s, three products of $\overline{\mathcal{Z}}_A$'s, the product of two $\mathcal{Z}^A$'s and $\overline{\mathcal{Z}}_A$ and the product of $\mathcal{Z}^A$ and two $\overline{\mathcal{Z}}_A$'s. The last six representations of (3.28) are obtained from (3.27) by multiplying $(\mathcal{Z}^4+\overline{\mathcal{Z}}_4)$. The last

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representation of (3.28) belongs to the $\mathcal{N}=2$ short graviton multiplet [11] and the other representations of (3.28) belong to the $\mathcal{N}=2$ long graviton multiplet. For $j=1,2,3$ cases in $\mathbf{27},\mathbf{7},\mathbf{1}$ of G_2, the structure of $\mathcal{N}=1$ SCFT operator is read off from the expression of hypergeometric function for $j=1,2,3$ cases (3.23).

4 Conclusions and outlook

We computed the KK reduction for spin-2 excitations around the warped 11-dimensional theory background that is dual to the $\mathcal{N}=1$ mass-deformed Chern-Simons matter theory with G_2 symmetry. The spectrum of spin-2 excitations was given by solving the equations of motion for minimally coupled scalar theory in this background. The AdS_4 mass formula of the KK modes is given by (3.21) and the corresponding wavefunctions on the 7-dimensional manifold are given by (3.22). The quantum number μ for G_2 representation and the KK excitation number j arise in this mass formula. We calculated the dimensions of the dual operators in the boundary SCFT via AdS/CFT correspondence and in table 1 we presented the summary of this work.

In the classification of [18], the massive multiplets for lower spins arise also. For example, the $OSp(1|4)$ supermultiplets with spin-$\frac{3}{2}$ components are massless gravitino multiplet($s_0=1$) with $D(\frac{5}{2},\frac{3}{2})$ denoted by Class 3 and massive gravitino multiplet($s_0=1$) with $D(E_0+\frac{1}{2},\frac{3}{2})$ where $E_0>2$ denoted by Class 4. The $\mathcal{N}=1$ massive gravitino multiplet $SD(E_0,1|1)$ with $E_0\geq 2$ originates from the $\mathcal{N}=2$ massive long gravitino multiplet. The massive $\mathcal{N}=8$ supermultiplet [25] at level n for spin-$\frac{3}{2}$ is given by SO(8) Dynkin labels $(n,0,0,1)\oplus(n-1,0,1,0)$. Definitely this provides the gauge theory operators dual to lower spin excitations. However, one should find out the right form for the perturbations that decouple from all other perturbations.

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A The differential operator, quadratic Casimir operator of G_2, and the generators of G_2

The differential operator acting on 7-dimensional ellipsoid is given by (3.5) and this can be written, from the metric (3.6) and the warp factor (2.8) with (2.11) and (2.7), in terms of angular coordinates as follows:

$$\begin{aligned}\left(\frac{2}{5}\right)^{-3/4}3^{-5/8}\mathcal{L} &= \frac{18}{5}\partial_\theta^2+\frac{108}{5}c_\theta s_\theta^{-1}\partial_\theta \\ &\quad+\frac{6}{5}\left(2+c_{2\theta}\right)s_\theta^{-2}\Big[s_{\theta_6}^{-2}\partial_{\theta_1}^2+4s_{\theta_1}^{-2}s_{\theta_6}^{-2}\partial_{\alpha_1}^2+4s_{\alpha_1}^{-2}s_{\theta_1}^{-2}s_{\theta_6}^{-2}\partial_{\alpha_2}^2+30s_{\theta_6}^{-1}c_{\theta_6}\partial_{\theta_6} \\ &\quad-8c_{\alpha_1}s_{\alpha_1}^{-2}s_{\theta_1}^{-2}s_{\theta_6}^{-2}\partial_{\alpha_2}\partial_{\alpha_3}-4s_{\alpha_1}^{-2}\left(c_{\alpha_1}^2-s_{\theta_1}^{-2}\right)c_{\theta_1}^{-2}s_{\theta_6}^{-2}\partial_{\alpha_3}^2-4c_{\theta_1}^{-2}s_{\theta_6}^{-2}\partial_{\alpha_3}\partial_{\theta_5} \\ &\quad+c_{\theta_1}^{-2}s_{\theta_6}^{-2}\partial_{\theta_5}^2+\partial_{\theta_6}^2+\left(1+2c_{2\theta_1}\right)s_{\theta_1}^{-1}s_{\theta_6}^{-2}s_{\theta_1}^{-1}\partial_{\theta_1}+4s_{\alpha_1}^{-1}c_{\alpha_1}s_{\theta_6}^{-2}s_{\theta_1}^{-2}\partial_{\alpha_1}\Big] \\ &= \frac{18}{5}\partial_\theta^2+\frac{108}{5}c_\theta^{-1}s_\theta^{-1}\partial_\theta+\frac{9}{40}\left(2+c_{2\theta}\right)s_\theta^{-2}\mathcal{C}_2, \end{aligned} \tag{A.1}$$

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where the quadratic Casimir operator (3.11) with (3.10) can be written as

$$\mathcal{C}_2 = \frac{16}{3}\left(\sum_{i,j=1,i\neq j}^{7}(X^i)^2\partial^2_{X^j} - 2\sum_{i,j=1,j>i}^{7}X^iX^j\partial_{X^i}\partial_{X^j} - 6\sum_{i=1}^{7}X^i\partial_{X^i}\right). \tag{A.2}$$

So it is obvious that the action of this $\mathcal{C}_2$ on the function $H(u)$ vanishes because the right hand side of (A.2) doesn't contain any differential operator on the variable X^8. The remaining parts of (A.1) can be written in terms of rectangular coordinates as follows:

$$\begin{aligned}\partial^2_\theta + 6c_\theta^{-1}s_\theta^{-1}\partial_\theta = {} & c_\theta^2 s_\theta^{-2}\sum_{i=1}^{7}(X^i)^2\partial^2_{X^i} - \sum_{i=1}^{7}X^iX^8\partial_{X^i}\partial_{X^8} + s_\theta^2\partial^2_{X^8} \\ & + \left(-1+7c_\theta^2\right)s_\theta^{-2}\sum_{i=1}^{7}X^i\partial_{X^i} - 7s_\theta c_\theta\partial_{X^8}.\end{aligned} \tag{A.3}$$

The first and fourth terms of (A.3) contribute to the terms in linear of H, the second and last terms contribute to the terms of H' and the third term contributes to the terms of H'' in (3.18).

The generators of G_2 can be chosen as real 7×7 matrices. The explicit realization of the generators was presented in [32]. The embedding of G_2 in the group SO(7) is generated by the 14 elements $T^a, a = 1, 2, \cdots, 14$:

$$T^1 = \begin{pmatrix} 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&-1\\ 0&0&0&0&0&-1&0\\ 0&0&0&0&1&0&0\\ 0&0&0&1&0&0&0 \end{pmatrix}, \qquad T^2 = \begin{pmatrix} 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&-1\\ 0&0&0&-1&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix},$$

$$T^3 = \begin{pmatrix} 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&-1&0&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&0&0&-1\\ 0&0&0&0&0&1&0 \end{pmatrix}, \qquad T^4 = \begin{pmatrix} 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&-1&0&0&0&0\\ 0&-1&0&0&0&0&0 \end{pmatrix},$$

$$T^5 = \begin{pmatrix} 0&0&0&0&0&0&0\\ 0&0&0&0&0&-1&0\\ 0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0\\ 0&0&-1&0&0&0&0 \end{pmatrix}, \qquad T^6 = \begin{pmatrix} 0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0\\ 0&0&0&-1&0&0&0\\ 0&0&1&0&0&0&0\\ 0&-1&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0 \end{pmatrix},$$

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$$T^7 = \begin{pmatrix} 0&0&0&0&0&0&0\\ 0&0&0&-1&0&0&0\\ 0&0&0&0&-1&0&0\\ 0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0 \end{pmatrix}, \tag{A.4}$$

and

$$T^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 0&0&0&0&0&0&0\\ 0&0&-2&0&0&0&0\\ 0&2&0&0&0&0&0\\ 0&0&0&0&1&0&0\\ 0&0&0&-1&0&0&0\\ 0&0&0&0&0&0&-1\\ 0&0&0&0&0&1&0 \end{pmatrix}, \tag{A.5}$$

$$T^9 = \frac{1}{\sqrt{3}} \begin{pmatrix} 0&-2&0&0&0&0&0\\ 2&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1\\ 0&0&0&0&0&-1&0\\ 0&0&0&0&1&0&0\\ 0&0&0&-1&0&0&0 \end{pmatrix}, \qquad T^{10} = \frac{1}{\sqrt{3}} \begin{pmatrix} 0&-2&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 2&0&0&0&0&0&0\\ 0&0&0&0&0&-1&0\\ 0&0&0&0&0&0&-1\\ 0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix},$$

$$T^{11} = \frac{1}{\sqrt{3}} \begin{pmatrix} 0&0&0&-2&0&0&0\\ 0&0&0&0&0&0&-1\\ 0&0&0&0&0&1&0\\ 2&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&-1&0&0&0&0\\ 0&1&0&0&0&0&0 \end{pmatrix}, \qquad T^{12} = \frac{1}{\sqrt{3}} \begin{pmatrix} 0&0&0&0&-2&0&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0\\ 2&0&0&0&0&0&0\\ 0&-1&0&0&0&0&0\\ 0&0&-1&0&0&0&0 \end{pmatrix},$$

$$T^{13} = \frac{1}{\sqrt{3}} \begin{pmatrix} 0&0&0&0&0&-2&0\\ 0&0&0&0&-1&0&0\\ 0&0&0&-1&0&0&0\\ 0&0&1&0&0&0&0\\ 0&1&0&0&0&0&0\\ 2&0&0&0&0&0&0\\ 0&0&0&0&0&0&0 \end{pmatrix}, \qquad T^{14} = \frac{1}{\sqrt{3}} \begin{pmatrix} 0&0&0&0&0&0&-2\\ 0&0&0&1&0&0&0\\ 0&0&0&0&-1&0&0\\ 0&-1&0&0&0&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0\\ 2&0&0&0&0&0&0 \end{pmatrix},$$

where the two matrices T^3 and T^8 generate the Cartan subgroup of G_2. There exist six SU(2) subgroups generated by the elements $(T^1, T^2, T^3), (T^4, T^5, \frac{1}{2}(\sqrt{3}T^8 + T^3)), (T^6, T^7, \frac{1}{2}(-\sqrt{3}T^8 + T^3)), (\sqrt{3}T^9, \sqrt{3}T^{10}, \sqrt{3}T^8), (\sqrt{3}T^{11}, \sqrt{3}T^{12}, \frac{1}{2}(-\sqrt{3}T^8 + 3T^3))$ and $(\sqrt{3}T^{13}, \sqrt{3}T^{14}, \frac{1}{2}(\sqrt{3}T^8 + 3T^3))$ from the fundamental commutation relations between these generators [33].

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